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CORRELATED QUADRATURES OF RESONANCE FLUORESCENCE AND THE GENERALIZED UNCERTAINTY RELATION

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Resonance fluorescence from a two-state atom has been predicted to exhibit quadrature squeezing below the Heisenberg uncertainty limit, provided that the optical parameters (Rabi frequency, detuning, laser linewidth, etc.) are chosen carefully. When the correlation between two quadratures of the radiation field does not vanish, however, the Heisenberg limit for quantum fluctuations might be an unrealistic lower bound. A generalized uncertainty relation, due to Schrödinger, takes into account the possible correlation between the quadrature components of the radiation, and it suggests a modified definition of squeezing. We show that the coherence between the two levels of a laser-driven atom is responsible for the correlation between the quadrature components of the emitted fluorescence, and that the Schrödinger uncertainty limit increases monotonically with the coherence. On the other hand, the fluctuations in the quadrature field diminish with an increasing coherence, and can disappear completely when the coherence reaches 1/2, provided that certain phase relations hold.

I. RESONANCE FLUORESCENCE

We consider a two-state atom, with excited state |e>, ground state |g>, and level separation ω_o, illuminated by an intense laser. The electric field at the position of the atom is assumed to have the form

$$\vec{E}_L(t) = E_o \, Re \, \vec{\varepsilon}_L \exp[-i(\omega_L t + \phi(t)] \quad , \tag{1.1}$$

where $\phi(t)$ is a stochastic phase which takes into account the laser linewidth. We shall assume that this is a diffusion process with independent increments. The driven atom will emit resonance fluorescence, and the positive-frequency part of the electric field of this radiation is

$$E^{(+)}(t) = \gamma d^{+}(t) \quad , \tag{1.2}$$

with $d^{+}(t) = |g\rangle\langle e|$ the atomic lowering operator, and γ an overall constant. The fluctuations in the amplitude (or phase) are measured by a homodyne detector with the driving laser as local oscillator. The slowly-varying operator under measurement is therefore[1]

$$E_\theta(t) = E^{(+)}(t)\exp[i(\omega_L t + \phi(t) - \theta)] \quad , \tag{1.3}$$

with θ the adjustable mixing angle of the detector. Essentially, the detector measures the variance of this quadrature operator, denoted by var(E_θ).

The fluctuations in $E_\theta(t)$ will be expressed in terms of the parameter r_θ, defined as

$$r_\theta = \mathrm{var}(E_\theta)/\langle E_\theta^2\rangle = 1 - \langle E_\theta\rangle^2/\langle E_\theta^2\rangle \quad . \tag{1.4}$$

Clearly, this parameter lies in the range $0 \le r_\theta \le 1$. For $r_\theta = 0$ we have var(E_θ) = 0, which corresponds to no fluctuations at all, and for $r_\theta = 1$ we have $\langle E_\theta\rangle = 0$. This case corresponds to a pure random phase of the field (like in a number state).

II. SQUEEZING

For Heisenberg's uncertainty relation we compare the quadrature field E_θ with the field $E_{\theta'}$, which is the same field except with a different mixing angle. Then the uncertainty relation can be written as

$$r_\theta\, r_{\theta'} \ge L_H^2 \quad , \tag{2.1}$$

with L_H the Heisenberg lower limit, given by

$$L_H = \frac{1}{2}\frac{|\langle[E_\theta, E_{\theta'}]\rangle|}{\sqrt{\langle E_\theta^2\rangle\langle E_{\theta'}^2\rangle}} \quad . \tag{2.2}$$

If we insert the expression for the quadrature field in L_H, it follows that the dependence on the mixing angles θ and θ' has the form

$$L_H \propto |\sin(\theta - \theta')| \quad , \tag{2.3}$$

and therefore we take $\theta' = \theta + n\pi$ with n as an integer. Then E_θ is said to be squeezed if

$$r_\theta < L_H \quad , \tag{2.4}$$

in view of (2.1).

The fluctuation parameter r_θ can be expressed in terms of matrix elements of the atomic density operator σ (either in the rotating frame or not). We find

$$r_\theta = 1 - 4|\sigma_{eg}(t)|^2 \cos^2\delta \quad , \tag{2.5}$$

with the angle δ equal to

$$\delta = \theta + \arg\gamma - \arg\sigma_{eg} \quad . \tag{2.6}$$

The Heisenberg lower limit can be written as

$$L_H = |n_g(t) - n_e(t)| \quad , \tag{2.7}$$

in terms of the populations n_e and n_g of the excited state and ground state, respectively. From (2.5) it then follows that the fluctuations in E_θ are minimum if the mixing angle θ is chosen such that δ becomes an integer multiple of π. This corresponds to maximum squeezing, given σ, because L_H does not depend on θ. For $\delta = n\pi$ the condition for squeezing becomes

$$1 - 4|\sigma_{eg}(t)|^2 < |n_g(t) - n_e(t)| \quad . \tag{2.8}$$

Whether squeezing occurs or not depends therefore on the populations of the atomic levels and the coherence between the levels.

III. STEADY STATE

When the atom has spent a sufficient amount of time in the laser field, its density operator will reach a steady state. The Rabi frequency of the transition is defined as

$$\Omega = E_o\left|\langle e|\vec{\mu}\bullet\vec{\varepsilon}_L|g\rangle\right|/\hbar \quad , \tag{3.1}$$

with $\vec{\mu}$ the dipole moment operator of the atom, and the laser linewidth λ will be parametrized through the combination

$$\eta = \tfrac{1}{2} + \lambda / A \quad , \tag{3.2}$$

where A is the Einstein coefficient for spontaneous decay. The detuning between the laser and the atomic resonance is

$$\Delta = \omega_L - \omega_o \quad . \tag{3.3}$$

In terms of these parameters, the absolute value of the coherence becomes[2]

$$|\sigma_{eg}| = \frac{1}{2}\Omega\frac{\sqrt{\Delta^2 + A^2\eta^2}}{\Omega^2\eta + \Delta^2 + A^2\eta^2} \quad , \tag{3.4}$$

and the difference in level population is

$$n_g - n_e = \frac{\Delta^2 + A^2\eta^2}{\Omega^2\eta + \Delta^2 + A^2\eta^2} \quad , \tag{3.5}$$

which equals the Heisenberg lower limit. The condition for squeezing, for $\delta = 0$, then becomes

$$(1-\eta)\Delta^2 > \eta^2(\Omega^2 + A^2(\eta - 1)) \quad , \tag{3.6}$$

as in Ref. (2). It is easy to see that for $\eta > 1$, which is $\lambda > A/2$, squeezing never occurs. The fluctuation parameter r is, for $\delta = 0$,

$$r = 1 - \Omega^2 \frac{\Delta^2 + A^2\eta^2}{[\Omega^2\eta + \Delta^2 + A^2\eta^2]} \quad . \tag{3.7}$$

For a given detuning Δ and laser linewidth λ, the fluctuations are minimum if we take the Rabi frequency, which is proportional to the laser power, equal to

$$\Omega^2 = (\Delta^2 + A^2\eta^2)/\eta \quad . \tag{3.8}$$

Then r becomes

$$r = 1 - 1/4\eta \quad , \tag{3.9}$$

which is independent of Δ. This minimizes for $\lambda = 0$, which gives $r = 1/2$ and $\Omega^2 = 2(\Delta^2 + A^2/4)$.

A convenient parametrization follows by introducing the new variable

$$\xi = \frac{\Omega^2}{\Delta^2 + A^2\eta^2} \quad , \tag{3.10}$$

dimensionless and proportional to the laser power. Then r and L_H can be written as

$$r = 1 - \frac{\xi}{(\xi\eta + 1)^2} \qquad (\delta = 0) \quad , \tag{3.11}$$

$$L_H = \frac{1}{\xi\eta + 1} \quad , \tag{3.12}$$

and the condition for squeezing becomes

$$\eta(\xi\eta + 1) < 1 \qquad (\delta = 0) \quad . \tag{3.13}$$

The dependence of r and L_H on ξ is shown in Fig. 1 for $\delta = \lambda = 0$. The difference $s = r - L_H$ is also shown, and a negative value of s corresponds to squeezing in the quadrature field below the Heisenberg uncertainty limit. Squeezing occurs for $\xi < 2$, and the maximum squeezing appears for $\xi = 2/3$, which gives $s = -1/8$. Notice that the minimum in r is located at $\xi = 2$, corresponding to $s = 0$. Hence, the best relative squeezing (minimum s) does not coincide with the smallest relative fluctuations in the field (minimum r).

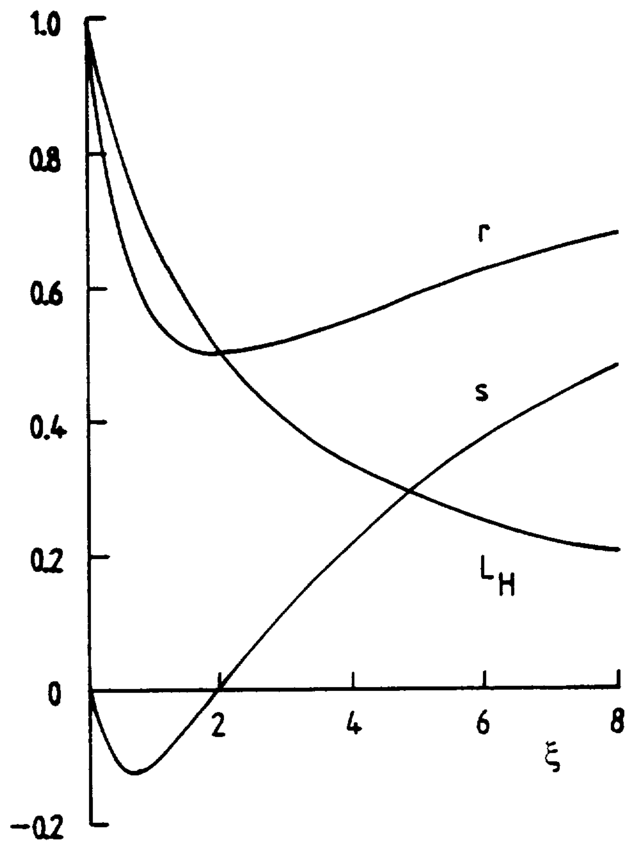

Fig. 1. Plot of parameters r, L_H, *and* s *as a function of* ξ.

IV. <u>SCHRÖDINGER UNCERTAINTY</u>

Heisenberg's uncertainty relation (2.1) sets a lower bound on the product of two variances. It is well known, however, that this is not the sharpest lower bound. A different uncertainty relation, due to Schrödinger, is[3]

$$r_\theta \, r_{\theta'} \geq L_S^2 \quad . \tag{4.1}$$

The Schrödinger lower bound is related to the Heisenberg lower bound by

$$L_S = \frac{1}{\sqrt{1-c^2}} L_H \quad , \tag{4.2}$$

where the correlation coefficient *c* is defined as

$$c = \frac{\frac{1}{2} < E_\theta E_{\theta'} + E_{\theta'} E_\theta > - < E_\theta >< E_{\theta'} >}{\sqrt{\mathrm{var}(E_\theta)\,\mathrm{var}(E_{\theta'})}} \tag{4.3}$$

It can be verified that c lies in the range

$$-1 \leq c \leq 1 \quad , \tag{4.4}$$

which then gives

$$L_S \geq L_H \quad . \tag{4.5}$$

This shows that the possible higher bound in Schrödinger's relation is due to the correlation between quadratures of the field with different values of θ. We shall always take $\theta' = \theta + \pi/2$, as before.

For resonance fluorescence the correlation coefficient can be expressed in terms of the matrix elements of the atomic density operator. We obtain

$$c = \frac{2|\sigma_{eg}|^2 \sin(2\delta)}{\sqrt{\{1 - 4|\sigma_{eg}|^2 \cos^2\delta\}\{1 - 4|\sigma_{eg}|^2 \sin^2\delta\}}} \quad , \tag{4.6}$$

showing that c is determined by the coherence between the levels only, and not by the populations of the atomic states. The relation between the Schrödinger limit and the Heisenberg limit then becomes

$$L_S = L_H \sqrt{1 + \frac{4|\sigma_{eg}|^4}{1 - 4|\sigma_{eg}|^2} \sin^2(2\delta)} \quad . \tag{4.7}$$

When δ is an integer multiple of $\pi/2$ or when the coherence is zero, we have $L_S = L_H$. When δ is not an integer multiple of $\pi/2$, the Schrödinger limit can become arbitrarily large when the coherence approaches 1/2 (any coherence, in absolute value, is smaller than 1/2 in a two-level system). Figure 2 shows the ratio L_S/L_H for $\delta = \pi/4$, and as a function of the atomic coherence.

In terms of the parameters ξ and η the relation (4.7) becomes

$$L_S = L_H \sqrt{1 + \frac{1}{4}\left(\frac{\xi}{\xi\eta + 1}\right)^2 \frac{\sin^2(2\delta)}{(\xi\eta + 1)^2 - \xi}} \quad . \tag{4.8}$$

The laser power that minimizes s for $\eta = 1/2$ and $\delta = 0$ is $\xi = 2/3$. Then the minimum value of s is -1/8 and the minimum of r is 5/8. Figure 3 illustrates the behavior of r, L_H, and L_S as a function of δ for these values of ξ and η.

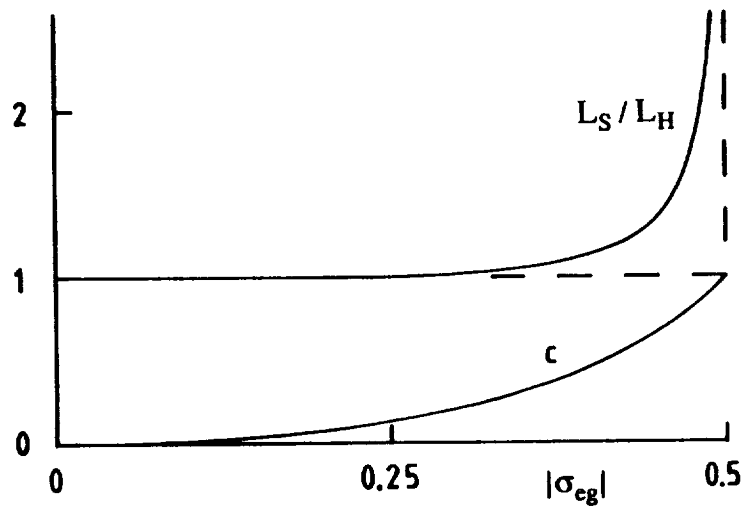

Fig. 2. Ratio L_S/L_H for $\delta = \pi/4$ as a function of the coherence.

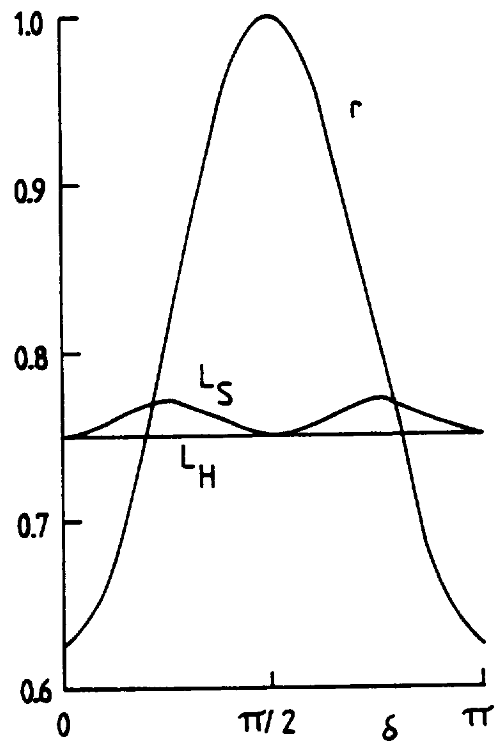

Fig. 3. Plot of the Heisenberg- and Schrödinger limits and the fluctuation parameter r as a function of δ.

V. CONCLUSIONS

We have studied the possibilities for squeezing in the quadrature components of resonance fluorescence from a two level atom. It was shown that the coherence between the two levels gives rise to a correlation between quadrature fields with a different mixing angle (in homodyne detection). This implies that the uncertainty limit on quantum fluctuations which is set by Schrödinger's relation can be considerably higher than the corresponding limit in Heisenberg's relation. It appears that in the steady state both limits are very close, as illustrated in Fig. 3. For pulsed-laser excitation, however, the coherence can approach its limiting value of 1/2, and this would increase the Schrödinger limit dramatically. Then the Heisenberg lower bound is an unrealistic lower limit, and squeezing should be defined with respect to the Schrödinger uncertainty relation.

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